Performance Analysis of Training and Blind Equalization Algorithms for Wireless Communication using 64-QAM

Asad Wahab[1,3], Sheeraz Ahmed [1,3], Nadeem Javed [2], Azhar Khan[1]

[1]Institue of Social Policy and Research (ISPaR), Peshawar, Pakistan
[2]COMSATS Institute of Information Technology, Islamabad, Pakistan
[3] Career Dynamics Research Centre, Peshawar, Pakistan

***Abstract* - Equalization is of utmost importance in all types of wireless communication systems. Every signal that is wirelessly transmitted must face the problem of multipath propagation. Multipath propagation results in spreading of individual symbols over time and hence results in Inter-Symbol Interference (ISI). The ISI causes the received signal to severely distort. In order to receive the original sent signal, the ISI must be eliminated from the received signal. In order to remove the ISI, a technique called as equalization is applied. In equalization we give the receiver certain information about the signal that is being sent. The receiver use that information to calculate the channel noise. The channel noise is then removed from the received signal in order to extract the original signal. There are two major types of equalization methods used in modern communication technology, the training based adaptive equalization algorithm and the blind equalization algorithm. In modern communication system, we have to look at various factors like voice quality, delay in signal transmission, distortion due to rapid movement of receiver or transmitter node or both. In this research, we will analyse the performance of these algorithm to figure out which algorithm suites the modern communication system and fulfil the demands of 4G and the upcoming 5G.**

Keywords – ***Wireless Communication, Equalization Algorithms, Training Equalization Algorithm, Blind Equalization Algorithm.***

I. INTRODUCTION

Today's 3G and 4G technologies are doing a great job of connecting the entire world and transferring trillions of bits every second; however, each technology comes with its problems. Our contemporary technologies are no alien to these concerns. From thermal noise to spectral efficiency there are multiple issues faced by these technologies which are hampering their performance. Some of the major issues include spectrum scarcity [1], multipath propagation that results in ISI, energy storage limitations and noises.

Among all the problems faced by the modern wireless communication systems around the world, the multipath propagation is a serious concern. A lot of work has been done on how to reduce the multipath and eliminate the ISI at the receiver's end.

Multipath results in ISI, which results in distortion of symbols. As stated in the paper [2], [3] ISI of the given plays a significant role in deciding about the performance of the communication system. In order to receive the undistorted signal at the receiver end, the ISI must be eliminated completely. ISI is considered as more destructive than any other noise available in then channel. The only way to effectively eliminate the ISI from a received signal is to use equalization. Equalization is considered as the best solution to the multipath propagation issue [4].

Equalization is the process through which we estimate the changes occurred to the received signal and then calculate those changes at the receiver and eliminate those changes in order to receive the signal. While there are several different equalization methods available, they can broadly be divided into two major types i) training/data aided equalization ii) blind equalization.

For any latest and modern communication system, there are a few parameters which are more important than they were for earlier generations of communication system.

A) Firstly, the conversion rate of any given signal in the process of equalization is of utmost importance. Considering a communication device which is constantly in motion from one point to another, there is a need to ensure lightning-fast signal conversion to avoid delay in the signal arrival at the receiver.

B) Secondly, there is a need to reduce the BER of the signal as much as possible in order to ensure smooth transfer of data. Since internet is mainly used on wireless devices nowadays, we cannot have much of BER in any given communication signal. Since ISI cause signal distortion and increase the chances of BER, there is a need of equalizer which can eliminate ISI and ensure a low BER.

C) Thirdly, there is a need to ensure that the feedback in the system does not consume too much of the bandwidth. In adaptive equalization algorithm, we send pilot signals to the receiver periodically in order to facilitate the receiver to extract the original signal. The pilot signal consume part of the bandwidth, we need to ensure that we achieve the process of ISI elimination while consuming as little bandwidth as possible [5].

There are multiple algorithms in use for equalization. In adaptive equalization the most common algorithms are Recursive Least Square (RLS) and Least Mean Square (LMS). [6] While in blind equalization the most common algorithms used are Godard's Constant Modulus Algorithm (CMA), Multi Modulus Algorithm (MMA) and Square Contour Algorithm (SCA) [7]. In this research, we have to compare these algorithms for any modern communication system using the advanced 16-QAM and 64-QAM constellation models in order to find the best algorithms in terms of ISI elimination, BER and bandwidth utilization that can be used in the upcoming and contemporary wireless technologies.

II. REVIEW OF EXISTING LITERATURE

As the technology progresses and latest communication generations like 4G and 5G are introduced, the need to revisit and revamp the equalization process and the algorithms become necessary. Here we'll have a look at the basic work done in the field of equalization to eliminate ISI.

A. Research based on ISI

In paper [2] the researchers have investigated the equalization process in communication channels with significant ISI. The paper proposes a design method for equalizer with a filtering error system I other to achieve minimum H- performance index even in an uncertain channel. The research went on to illustrate two practical design of the model to prove the effectiveness and practicality of the proposed method.

In paper [3] the authors have studies the statistical distribution of a single user Ultra-Wideband system in present of ISI and the AWGN. The model is simulated by collecting the data of the conditional probability density function at the output in the receiver block. The results demonstrate that the Middleton Class-A distribution model work efficiently in the Line-Of-Sight (LOS) but does not work very efficiently in a Non-LOS model.

B. Research based on Training Algorithm

In paper [8], the author has given a brief review of the adaptive equalization in context of solving the issue of ISI in communication system. The paper also introduces the transversal equalizers along with some practical structures for adaptive equalizers. The paper explains through mathematical models and figures, the entire process of adaptive equalization.

The authors of paper [6] have compared the performance of ZF, LMS and RLS Algorithms. The paper concludes that the LMS performs better than the ZF algorithm in terms of the BER. While RLS provides channel equalization by reducing the channel effects, however, RLS' performance can further be improved using neural network equalization.

Paper [9] analyzes the result of LMS algorithm applied on two communication channels. The authors also compared the simulation results of LMS and Constant Modulus Algorithm (CMA) for the test channel.

Paper [10] determines the linear precoder in an effort to reduce the BER at a medium-to-high SNR with ZF equalization. The design suggested in this paper is developed to eliminate the inter-block interference through Zero-Padding (ZP) and Cyclic Prefix (CP). The paper shows that minimum BER precoder in both ZP and CP provide significantly low error rate than common block schemes.

C. Research based on Blind Equalization

Paper [11] compares the performance of algorithms for blind equalizations i-e the CMA, Stop-and-go decision directed algorithm and Wei Rao's modified CMA for QAM constellation across linear band-limited channel. The paper comes to a conclusion that stop-and-go algorithm has a better performance than others when compared for Mean Square Error (MSE) and convergence rate.

In paper [12] authors have proposed an MMA algorithm which is a modified version of CMA based on the cost function. The paper eliminate the CMA's separate system for phase recovery. The authors have found that the proposed algorithm provides better data rate and a lower Bit Error Rate (BER).

In paper [13], semi-blind / blind equalizers are studied for a fading MIMO channel. The paper defines a composite channel for each equalizer and then several algorithms based on channel capacity. Paper concludes that in Ricean Environment semi-blind / blind algorithms performs better than training equalizers, but in Rayleigh channels, training methods outperform the rest.

D. Research on Equalization in Latest Technologies

Paper [14] studies the use of extended bandwidth usage in VLC through the use of phosphorescent white LED. This research proposes a post-equalization circuit which contains one active and two passive equalizers. A bandwidth of 151 MHz is achieved in the given VLC communication model using post-equalization circuit and blue-filtering. The bandwidth allows OOK-NRZ transmission of up to 340 Mbps. The VLC model selected works at 43 cm using a 1 Watt LED with a BER staying below $2x10^{-3}$.

Researcher of Paper [15] has practically demonstrated using an experiment a 3.24Gbps VLC communication system using 512 QAM SC-FDE with the use of an RGB-LED with bandwidth of 10 MHz and 3-db. Three wavelength channels were utilized during the experiment and the BER of all the three remained below the threshold of pre-FEC set at $3.8x10^{-3}$.

III. COMPARATIVE ANALYSIS OF EQUALIZATION ALGORITHMS

There are several different equalization methods available, they can broadly be divided into two major types:

A. Training based Adaptive Equalization

In training based data aided equalization technique, a chunk of data called as training/pilot signal is introduced to the receiver which helps the receiver learns the channel values and then use that data for channel estimation and ISI elimination. The training based equalization method has a quick conversion rate, better efficiency and has simple application. This method is considered best for environment where fast fading is required with high Doppler spread and little coherence time. The downside to these equalizers; however, is that they constantly need pilot signals. The constant transmission of the training sequence consumes a lot of bandwidth which is a significant downside. In GSM around 18% of the bandwidth is consumed by the training sequences that are periodically sent to the receiver [5]. There are multiple training algorithms that can be used in a training-based adaptive equalizer e.g. LMS [6] and RLS [7].

B. LMS Equalization

LMS is the most common form of adaptive equalization. The LMS uses stochastic gradient descent for updating the equalizer weights during its operation. For any complex channel gain, the complex version of LMS equalizer is utilized and is given by

$$z(k) = \omega^{\tau}(k)x(k) \quad (1)$$

z(k) represents the output of the given adaptive equalizer and is equal to the product of received signal and weight of the given equalizer.

$$e(k) = s(k) - z(k) \quad (2)$$

Error estimation is represented in eq. 2 by e(k), while s(k) shows the desired signal. Subtracting the equalizer gain z(k) from the desired signal s(k) would give us the error estimation of the system. The weight update of the system can be calculated using the following

$$w(k + 1) = w(k) + 2\mu e * (k)x(k) \quad (3)$$

Eq. 3 include * as the complex conjugate while μ represents the step size which is related to the convergence rate of the equalizer. The process of finding the perfect μ value for efficient convergence is a tedious process and several numeral values are tried in this research in order to come up with the optimal convergence. The equation update the tap which then further changes the weight of the filter until the LMS equalizer is able to give an optimal convergence rate.

C. Recursive Least Square

Recursive Least Square (RLS) algorithm is another type of adaptive algorithm which has a higher computational complexity than its counterpart LMS. The working of an RLS equalizer can be calculated through the following formula

$$u(k) = \psi_{\lambda}^{-1}(k - 1)x(k) \quad (4)$$

Ψ is used in equation 4 for reducing computational complexity. Ψ show a diagonal matrix with diagonal entries with value of 1, λ, λ2… Matrix inversion lemma is used in order to create a recursion in ψ_{λ}^{-1}. Input vector is shown by *x(k)*

$$x(k) = \frac{1}{\lambda + x^{H}(k)u(k)}u(k) \quad (5)$$

Gain computed in the equation 5 depends on the value of λ. While both equations 4 and 5 are jointly used to calculate K(k) the gain vector. The λ which stands for the forgetting factor has a value near 1. λ, the weighting factor, provide less weightage to earlier samples and more to new ones and ignore the earlier ones.

$$\hat{z}_{k-1}(k) = \hat{w}^{H}(k - 1)x(k) \quad (6)$$

Equation 6 shows the input signal filtering in RLS equalizer. The $\hat{z}_{k-1}(k)$ is the output of the equalizer while $\hat{w}(k)$ show the updated weights. Just like the LMS, the error estimation in RLS equalizer is computed through the following

$$\hat{e}_{k-1}(k) = s^{*}(k) - \hat{z}_{k-1}(k) \quad (7)$$

$\hat{e}_{k-1}(k)$ represents error which is calculated using the $s^{*}(k)$ that represents the desired signal and the $\hat{z}_{k-1}(k)$ which is the output of the equalizer.

$$\hat{w}(k) = \hat{w}(k - 1) + K(k)\hat{e}_{k-1}(k) \quad (8)$$

Equation 8 is the tap update equation for the RLS equalizer. Gain K(k) and e(k) is multiplied in order to find the tap change for K^{th} iteration of the equalizer.

$$\psi_{\lambda}^{-1}(k) = \lambda^{-1}(\psi_{\lambda}^{-1}(k-1) - K(k)[x^{H}(k)\psi_{\lambda}^{-1}(k-1)] \quad (9)$$

Equation 9 is used for updating ψ_{λ}^{-1}.

D. Blind Channel Equalization

The other equalization method is blind channel equalization. This technique is very handy when it comes to a system with one transmission point and multiple receiving node. If we send a training signal to each of the receiving node periodically, we will be consuming a lot of bandwidth. In order to solve this issue, we use blind equalization method so we do not have to send any training signal. The equalizer only needs to know about the mapping technique of the signal and then estimate channel effects accordingly. There are multiple algorithms including MMA and SCA [16].

E. Multi-Modulus Algorithm

Multi-Modulus Algorithm (MMA) is the advanced form of the old CMA algorithm. In the old fashioned CMA, the real and imaginary parts of an equalizer's output had to be separated. In MMA as well, both the real and imaginary parts are separated in order to get the cost function, which can be mathematically presented as

$$J_{MMA} = \boldsymbol{E}\{(|z_{kr}|^{p} - R_{MMA}^{p})^{2} + (|z_{ki}|^{p} - R_{MMA}^{p})^{2}\} \quad (10)$$

In equation 10 **E** denotes expectation operator, z_{kr} denotes the real part and z_{ki} denotes the imaginary part of the output of equalizer for k^{th} value. 'p' mentioned in the equation denotes an integer necessary for the calculation.

$$R_{MMA}^{p} = \frac{E\,|s_{kr}|^{2p}}{E\{|s_{kr}|^{p}\}} = \frac{E\,|s_{ki}|^{2p}}{E\,|s_{ki}|^{p}} \quad (11)$$

R_{MMA}^{p} is known as Goddard's statistical constant. While S_{kr} is the real part of the equation while S_{ki} is the imaginary part of the equation. In a complex constellation, the equalizer dispersion is visible around 4 pints for MMS cost function ($\pm R_{MMA} \pm jR_{MMA}$) and can be denoted as the addition of two cost functions. We can increase the performance of the MMA equalizer by increasing the value of p at the cost increase complexity. For the sake of simplification and practicality, we choose 2 as the value of p. In order to update the weight of the MMA based equalizer, we use the following formulae

$$e_{k} = z_{kr}\,|z_{kr}|^{p-2}(|z_{kr}|^{p} - R_{MMA}^{p}) + jz_{ki}\,|z_{ki}|^{p-2}(|z_{ki}|^{p} - R_{MMA}^{p}) \quad (12)$$

MMA equalizer is known for recover the distortion in the phase of the signal much efficiently.

F. Square Contour Algorithm

The Square Contour Algorithm (SCA) is based on the constellation of the received signal. The traditional CMA minimize the dispersion of the output of the equalizer considering a circular constellation. The SCA, on the other hand minimizes the equalizer output using a square constellation and also recover the entire phase shift incurred during the transmission of the signal. The cost function for SCA based equalizer can be written as

$$J_{SCA} = E\{(|z_{kr} + z_{ki}| + |z_{kr} - z_{ki}|)^{p} - R_{SCA}^{p})^{2}\} \quad (13)$$

Just like the MMA, in SCA too, the real and imaginary parts are separately considered. In equation 13, J is the equalizer output of the SCA based equalizer. E represents expectation operator while p is an integer. The real and imagery parts of the equalizer's output are denoted by z_{kr} and z_{ki} respectively. R in the equation represents a constant whose value depends on the type of constellation used in the wireless communication system. As we have

$$|z_{kr} + z_{ki}| + |z_{kr} - z_{ki}| = 2\max\{|z_{kr}|, |z_{ki}|\} \quad (14)$$

The zero-error contour for the SCA equalization based system can be written as

$$\max\{|z_{kr}|, |z_{ki}| = \frac{R_{SCA}}{2} \quad (15)$$

Equation 15 represents a square with center as its origin. The error $e_{k,SCA}$ can be mathematically written as

$$e_{k,SCA} = ((|z_{kr} + z_{ki}| + |z_{kr} - z_{ki}|)^p - R_{SCA}^p)(|z_{kr} + z_{ki}| + |z_{kr} - z_{ki}|)^{p-1} \times (\text{signum}[z_{kr} + z_{ki}](1 + j) + \text{signum}[z_{kr} - z_{ki}](1 - j)) \quad (16)$$

Where the R_{SCA}^p represents the constant as per the constellation an can be mathematically showed as

$$R_{SCA}^p = \frac{E\{(|s_{kr} + s_{ki}| + |s_{kr} - s_{ki}|)^p . Q\}}{E(Q)} \quad (17)$$

Q in the given equation can be mathematically explained as

$$Q = (|s_{kr} + s_{ki}| + |s_{kr} - s_{ki}|)^{p-1}(\text{sgn}[s_{kr} + s_{ki}](1 + j) + \text{signum}[s_{kr} - s_{ki}](1 - j))\, s_k^* \quad (18)$$

Where * represents a conjugate while the s_{kr} and s_{ki} stand for real and imaginary parts, respectively.

IV. Simulation Results and Discussion

Based on the mathematical derivations, MATLAB simulation was run, here are the results.

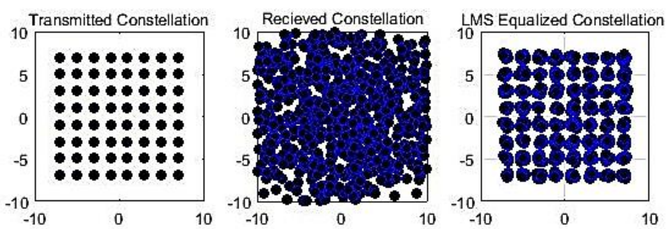

Figure 6.1: LMS Equalization algorithm in 64 QAM constellation

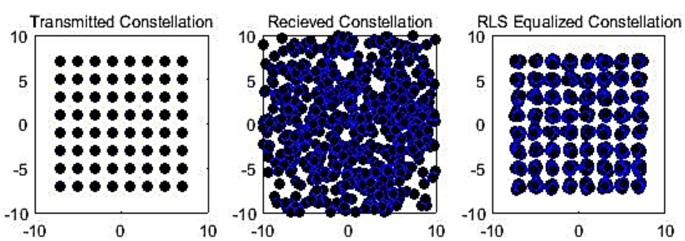

Figure 6.2: RLS Equalization algorithm in 64 QAM constellation

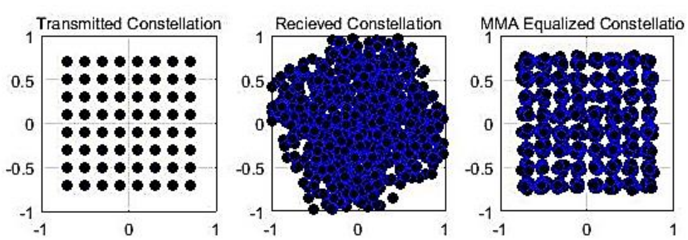

Figure 6.3: MMA Equalization algorithm in 64 QAM constellation

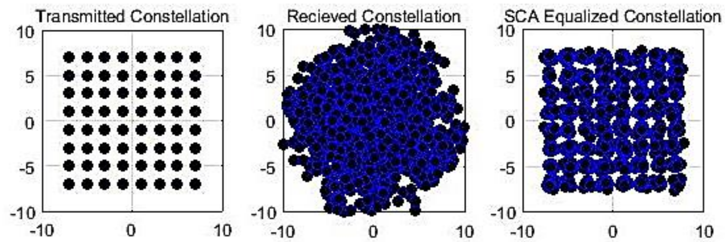

Figure 6.4: SCA Equalization algorithm in 64 QAM constellation

In the plots above, the left side represents the original transmitted signal to the receiver, the central figure represents the distorted and phase shifted signal due to the channel values and the AWGN present in the medium. This results in a distorted and shifted signal at the receiver with ISI. Now in order to extract the original signal, we applied each of the algorithms and the figure at right in each of the plots is the equalized signal.

A. BER Comparison of Equalization Algorithms for 64-QAM

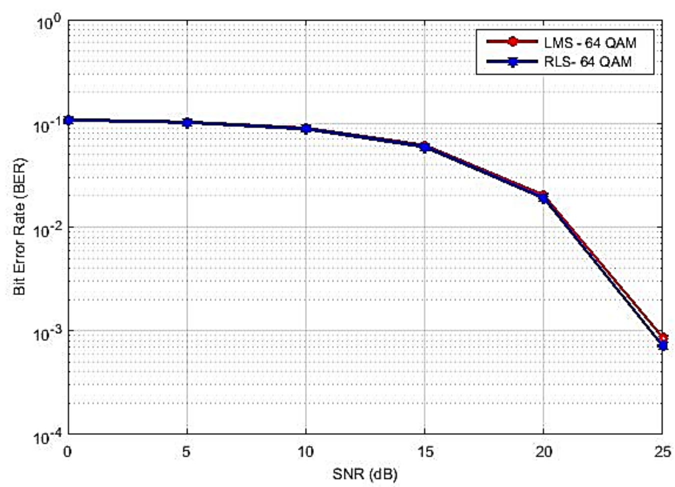

Figure 6.5: BER comparison of LMS and RLS Algorithm for a 64-QAM constellation

In order to compare the BER of LMS and RLS for 64 QAM constellation, we need to compare the two graphs. We can see that the values remain almost the same till 20db, above that the RLS start exhibiting a slight improvement in terms of BER as the BER is reducing significantly at around 25db and if we go beyond that the performance of RLS will get even better. Hence we can safely conclude that for low intensity signal, both LMS and RLS have the same BER performance, however, for higher intensity signal, RLS is preferred for its better performance in terms of BER.

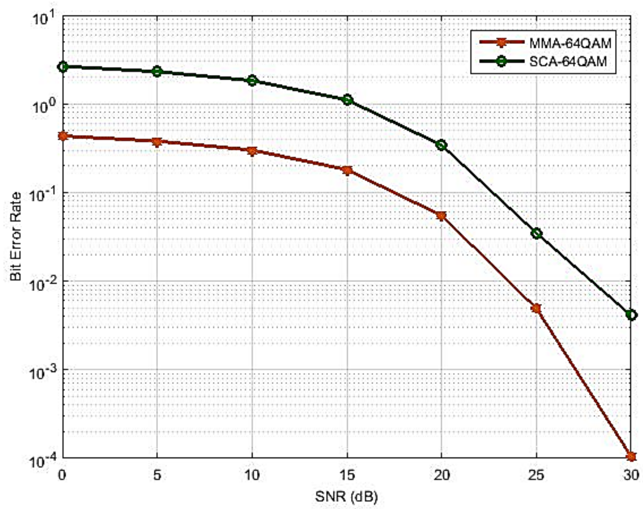

Figure 6.6: BER comparison of MMA and SCA Algorithm for a 64 QAM constellation

The plot above depicts the comparison of MMA and SCA for 64 QAM constellation and we can see the very significant difference in the performance of the two blind equalization Algorithm. For any given value of SNR, the BER of MMA is much better than that of SCA. However, there is a constant difference between the values of the two, at 25db the value of the MMA start getting even better. At 30db, we can witness a significant improvement. Hence it can be concluded that the performance of MMA is comparatively better than the SCA for 64 QAM constellation.

B. ISI Residual Comparison of LMS and RLS for 64-QAM

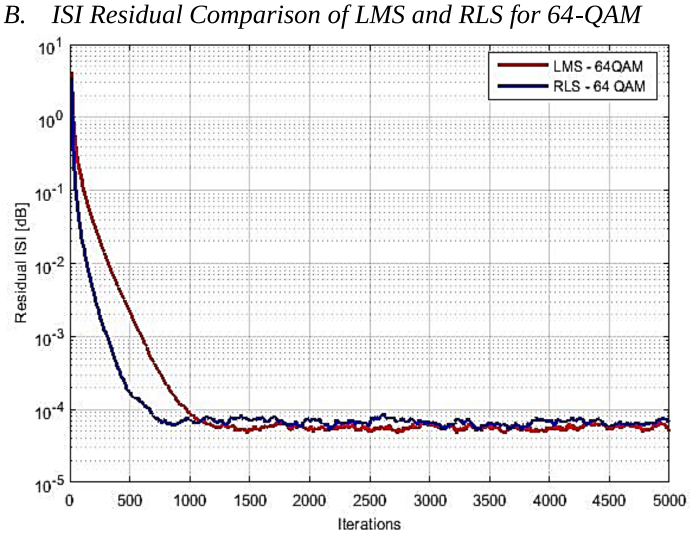

Figure 6.7: ISI residual comparison of LMS and RLS Algorithm for a 64 QAM constellation

In plot above, we can see the ISI residual comparison of the two training based algorithms for 64 QAM constellation. The RLS in the plot above has a better convergence rate than the LMS and at around 700 iterations, the RLS can be see stabilizing. While the LMS takes long in convergence and needs around 1000 iterations to stabilize. Hence it can be concluded that utilization of LMS for communication between two fast moving nodes will result in a delay. However use of RLS can eliminate that problem, but it will cost more hardware and complex calculation to achieve this.

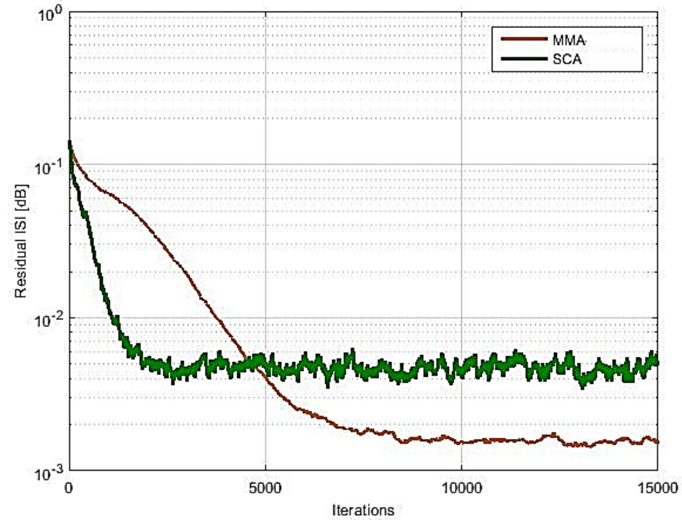

Figure 6.8: ISI residual comparison of MMA and SCA Algorithm for a 64 QAM constellation

The plot above represent the comparison between MMA and SCA for ISI residual in a 64 QAM constellation. As we can see, the SCA has a quick convergence and requires only 400 iterations to retain a constant and stable residual ISI value. While in comparison the MMA requires around 7000 iterations to stabilize. However, the performance of MMA is pointedly better than that of the SCA but it comes at the cost of delay in the signal. If the system of communication can bear the delay then MMA is the best option to go for. However, if quick convergence is of utmost priority and no compromise can be made on delay then SCA is the first choice to be used.

C. MSE Comparison of LMS and RLS for 64 QAM

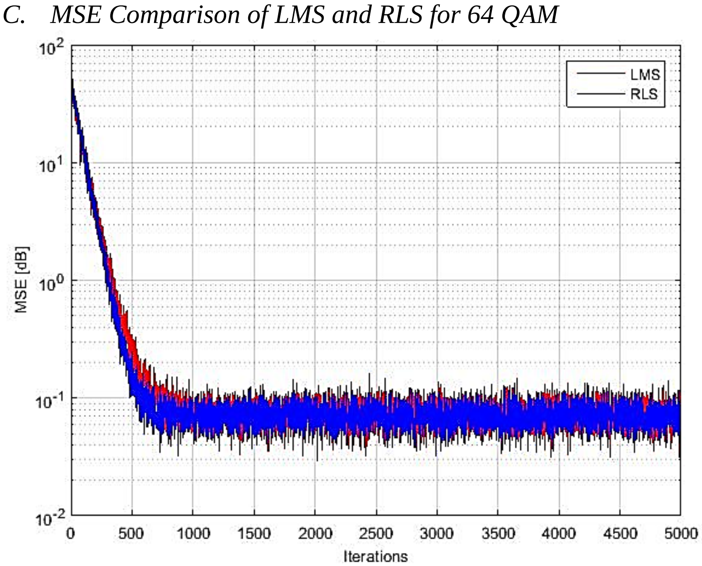

Figure 6.9: MSE comparison of LMS and RLS Algorithm for a 64 QAM constellation

For 64 QAM constellation. The MSE performance of LMS and RLS has very little difference once they stabilize. However, the major difference between these two training equalization algorithms lies in its convergence rate. The LMS algorithm takes a while in stabilizing and requires around 750 iterations to stabilize while the RLS requires 600 iterations. Hence RLS has better convergence rate and similar MSE performance at static conditions.

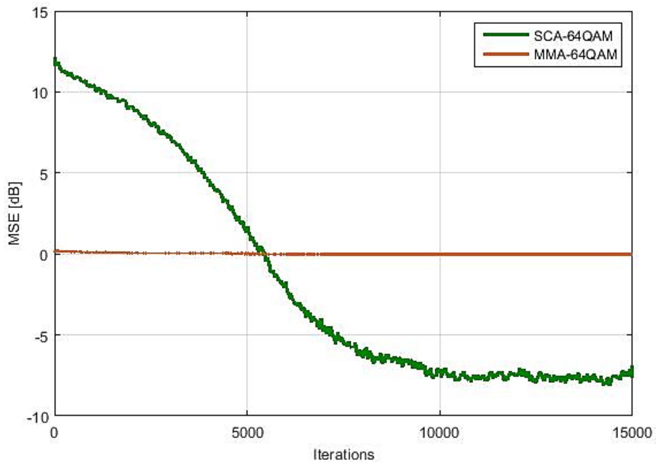

Figure 6.10: MSE comparison of MMA and SCA Algorithm for a 64 QAM constellation

The plot above represents the comparison of MMA and SCA blind algorithms for a 64 QAM constellation. The convergence rate of MMA is very high than that of SCA, however; the performance of SCA in terms of MSE is far better than the MMA. It takes around 200 iterations for MSE to stabilize while SCA takes around 8000 iterations before stabilizing. Hence for a static system using 64 QAM, SCA should be the first priority. It will initially take a while to stabilize, however; once stabilized the MSE of the signal will remain very little and almost negligible hence resulting in smooth transfer of signal.

X. Conclusion

All the existing and upcoming wireless communication technologies need to ensure smooth communication during static mode and a flawless communication between two moving nodes. Since ISI is one of the major problems in ensuring these two, the algorithms that we compared can solve the issues. In training based equalization algorithms, the LMS is comparatively simple and provide quick convergence, the RLS on the other hand has better performance when it comes to BER, MSE and residual ISI, however; due to the complexity involved and slow convergence, the RLS can be preferred for wireless systems with relatively static nodes. In blind equalization algorithm, the MMA general has a better performance than the SCA, however; like RLS the SCA needs complex calculations and at times provide better results in terms of ISI but is more costly. To sum it up, the equalization algorithms should be used in cases to case basis or a dynamic wireless communication system needs to be utilized which can switch between these algorithms rapidly based on the environment in order to ensure smooth flow of communication.

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